Multidimensional Condition Monitoring of Machines in Nonstationary Operation[1]

Czesław CEMPEL[2], Maciej TABASZEWSKI[3]

Summary

Many critical mechanical systems operate in a nonstationary regime (*load*), and many observed symptoms of its condition depend in a some way on a system load and/or environmental conditions. Hence the condition monitoring of a such systems ought to have some possibility of rescaling of observed symptoms to a **standard load** condition. This paper shows such a possibility of a symptoms rescaling in application to multidimensional vibration condition monitoring. It is shown on some real example of vibration condition monitoring, that rescaling of symptoms can make more reliable the assessment of current system condition, as well as its prognosis.

Key words: symptom observation matrix, multi fault observation, singular value decomposition (**SVD**), symptom rescaling.

1. Introduction

The idea of multidimensional condition monitoring of critical machines, with the many symptoms[4] concurrently observed and the application of singular value decomposition (**SVD**), was formulated firstly in the paper [1]. Here the concept of symptom observation matrix (**SOM**) was used in order to extract by SVD the new fault related **generalized symptoms,** or fault indices. In the latest approach to multidimensional diagnostic observation, the principal component analysis (**PCA**) is used to extract fault related information (*see for example*[12], [13]). But in case of using PCA the small faults may be omitted due to the squaring effect of singular values. The latest development of this concept using SVD is presented in the last papers of the first author [2], [4]. In another development, it is known that some symptoms of condition may depend on working and environmental condition of the machine, what was described in some papers of the first author as **logistic vector** $\boldsymbol{L}$ concept [2,p59]. For the final purpose of the machine diagnosis we need also some method of symptom limit value $\boldsymbol{S_l}$ determination, what can be done by the use of **symptom reliability** concept [9]. But until now, there was no common connection and application of these three concepts in machine condition monitoring area.

As it was mentioned **SVD** method allows as to extract from the data of **SOM,** a few generalized symptoms related to the main faults $\boldsymbol{F_i(\theta)}$ evolving in operating machine. From the other side, by means of symptom reliability $\boldsymbol{R(S)}$ [9], and generalized **symptom life curve** obtained from **SVD** we can determine symptom limit value $\boldsymbol{S_l}$. For machines working in stationary load condition it is enough to design the condition monitoring system. But in case of nonstationary operation we need to use the dependence of some symptoms on logistic vector $\boldsymbol{L(l,..)}$, mainly on system load l in a first approach. Such is the main idea of this paper, and the goal is to extend the applicability of above to the range of nonstationary operation of machines, basing on existing programs and results of multidimensional condition monitoring.

2.Rescaling of symptoms in the symptom observation matrix

---

[1] Paper partially presented to ICSV12 Lisbon-05, [11].

[2] Corresponding author

[3] Poznań University of Technology, Poznań, **Poland**, email: czeslaw.cempel@put.poznan.pl maciej.tabaszewski@put.poznan.pl, fax +48.61.6652307

[4] **Symptom** is the measurable quantity, taken from the phenomenal field of the object, and proportional to the condition we are looking for.

Let us take into consideration S_{pn} one of the component of the observation vector ***S*** of the symptom observation matrix, having the ordering number n. It is known already that some diagnostic symptoms exhibits sensitivity to the value of logistic vector ***L***, hence its observation made in the life time θ_p will depend on both variables, namely; $S_{pn}(\theta_p, L_p)$.

If we know the standard value of the operational load l_o, where the most of the observations has been made, we can perform the symptom rescaling operation. By means of Taylor series expansion in the vicinity of normative load l_o, and assuming the load deviation Δl_p at the life time θ_p, one can get the approximation as below

$$\mathbf{S}_{pn}(\theta_p, L_p) = S_{pn}[\theta_p, L_o(1 + \frac{\Delta L_p}{L_o})] \cong (1 + \frac{\Delta L_p}{L_o})\mathbf{S}_{pn}(\theta_p, L_o), \tag{1}$$

what is equivalent in a first approach to **multiplicative** influence of the logistic vector on the observed symptom value.

In this way, intending to bring ***n***-th observation to the condition with standard load l_o, it means to the rescaled value $S_{pn}(\theta_p, l_o)$, we should multiply it by the reverse rescaling coefficient taken from (1) in the form

$$l_p = (1 + \frac{\Delta l_p}{l_o})^{-1} \quad . \tag{2}$$

It means, that for positive value of the load increment $\Delta l > 0$ with respect to standard load l_o we should divide it by the coefficient greater than one. And vice versa, for the negative increment of the load ($\Delta l < 0$ - *decreased load*) at the time of symptom observation we should divide it by the coefficient less than one.

So much if we concern with rescaling of one symptom *S* only. Assuming now that the similar behavior can be a property of other symptoms[5] (*columns*) in **SOM**, we can use (*in the first approach*) the same coefficient for all observations taken in a time θ_p, it means to the whole row *p* of **SOM**.

Let us assume now that our matrix notation will be the same as in a previous papers (*see for example*[2]), with the **SOM** denoted as $O_{pr} = [S_{mn}]$. This notation means; with the maximal number of rows *p*, with current row number (*observation*) *1, ...m*, and the number of symptoms ***r*** (*columns*), succeeding from; *1,...n*. Hence, doing now rescaling of **SOM,** we should multiply the whole row of the matrix by rescaling coefficient (2). It follows from the matrix calculus [5,p26], that it should be done by left hand side multiplication ($*$) of the diagonal rescaling matrix having, a rank equal to number of rows of current dimension (*m*) of **SOM** matrix. In the light of the above, the rescaling operation of **SOM** to the standard load condition l_o can be done as below

$${}^{R}O_{pr} = L_{pp} * O_{pr} = [L_{mm}] * [S_{mn}] \text{ , } m=(1,p), \tag{3}$$

where the left hand side superscript *R* means rescaling, and a square bracket over matrices means current making of rescaling operation by the matrix L_{mn}. In the most general case the logistic rescaling matrix can have different multiplying coefficient for every row, as below

$$L_{mm} = Diag\,(1 + \Delta l_m / l_o),\ m = (1,p), \tag{4}$$

and if the rescaling operation for the given row is not needed, then the respective row in the logistic matrix becomes unity.

It seems to be reasonable, that the same concept of rescaling can be applied to another components of a logistic vector ***L***, coming from the environment for example. Just to remind, that for the machines the components of ***L*** can also describe another different features such as

[5] The detailed difference in sensitivity between symptoms can be expressed by some coefficients, if needed.

the quality of design, the quality of production, quality of maintenance, index of foundation stiffness, and other (*see for example* [6]). The same can be done to variable environmental conditions, such as the gust of the wind in aerogenerator, the immersion and the pitch of the ship, etc. It is obvious now, that rescaling concept can be used not only to the load l of machine, and environmental conditions, but also for the quality o maintenance operations, where one can make rescaling of initial machine observations in **SOM**, just after its start up.

3. Fault information extraction and interpretation of rescaled symptom observation matrix

Just to remind ourselves, we are observing the condition of complex mechanical system, which can exhibit its wearing processes by means of a few dominating faults[6], described by generalized symptoms $\boldsymbol{F_t(\theta)}$, ***(t=1,2,...)***. Trying to catch and describe this multidimensional degradation process of the machine we observe multidimensional symptom space by means of symptom observation vector $\boldsymbol{S}$, and the successive life time realizations of this vector for $\boldsymbol{\theta_n}$ ***(n=1 2, ...)*** gives us **SOM**. As we have mentioned already, the best method to extract these information from rectangular **SOM** is the application of singular value decomposition (**SVD**), similar to the older method principal component analysis (**PCA**), (*for example [12]*). It is good to know that, the last one is less sensitive to small energy components of **SOM** due to matrix multiplication inherent by definition in **PCA** [6]. In our previous papers the original **SOM** was transformed column wise by centering and normalizing to the initial value ($\boldsymbol{\theta=0}$) of each symptom (*column*). Hence our rescaled **SOM** can be written now as below;

$$ {}^{R}\boldsymbol{O_{pr}} = L_{pp} * O_{pr} = [L_{pp}] * [S_{nm}], \qquad S_{nm} = \frac{\mathbf{S}_{nm}}{\mathbf{S}_{0m}} - 1, \tag{5} $$

where the bold letters means primary measured symptom values before its transformation.

The **SVD** procedure for any rectangular matrix gives us left hand and right hand side singular vectors $\boldsymbol{U_i}$, $\boldsymbol{V_i}$, and singular values $\boldsymbol{\sigma_i}$, as below [7], [8];

$$ {}^{R}\boldsymbol{O_{pr}} = U_{pp} * \Sigma_{pr} * V_{rr}^{T}, \qquad (\text{T- transposed matrix}), \tag{6} $$

Where $\boldsymbol{U_{pp}}$ square matrix of left hand side singular vectors, and $\boldsymbol{V_{rr}}$ square matrix of right hand side singular vectors, and $\boldsymbol{\Sigma_{pr}}$ diagonal matrix of singular values as below;

$$ \boldsymbol{\Sigma_{pr}} = Diag\,(\sigma_1, \ldots, \sigma_l), \ \ and \ \ \sigma_1 > \sigma_2 > \ldots > \sigma_u > 0, \tag{7} $$
$$ \sigma_{u+1} = \ldots \sigma_l = 0, \quad l = max\,(p, r), \quad u = min\,(p, r). $$

The diagnostic interpretation of **SVD** method elaborated so far in some papers leads us to two quantities obtained from the above. The **generalized fault symptom** of order $\boldsymbol{t}$ which shows the **life time profile** of this fault $P_t\,(\theta)$;

$$ SD_t = {}^{R}O_{pr} * v_t = \sigma_t \cdot u_t \ \sim P_t\,(\theta), \quad t = 1, \ldots, u. \tag{8} $$

The second quantity is the energy norm of the above, so it can represent the cumulative advancement of given generalized fault

$$ Norm\,(SD_t) \equiv // SD_t // = \sigma_t. \sim F_t\,(\theta), \quad t = 1, \ldots, u. \tag{9} $$

[6] Faults can be different physically, having different spatial location in a machine, and even can be independent each other, at least at the beginning of wearing process.

Of course, when tracing the fault evolution in the operating machine, both quantities will depend on a life time of the system $\boldsymbol{\theta}$, so we will have; $\boldsymbol{SD_t(\theta)}$ and $\boldsymbol{\sigma_t\,(\theta)}$. Finally, it seems to be good to
monitor cumulative wear, so the advancement of all generalized faults $F_t\,(\theta)$ in a machine, by means of **summation quantities** (8) as given below;

$$SumSD_i(\theta) = \sum_{i=1}^{z} SD_i(\theta) = \sum_{i=1}^{z} \sigma_i(\theta)\cdot u_i(\theta) = P(\theta)\,, \tag{10}$$

$$Sum\,\sigma_i(\theta) = \sum_{i=1}^{z} \sigma_i(\theta) \sim \sum_{i=1}^{z} F(\theta)_i = F(\theta)\ . \tag{11}$$

As it was shown in [4] such interpretation of generalized fault indices in operating system obtained by **SVD** seems to be correct, what allows us to use **symptom reliability** concept $\boldsymbol{R(S)}$ for determination of generalized symptom life curve $\boldsymbol{S_u(\theta)}$ and symptom limit value S_l **.** Adding now **rescaling** capabilities to the above theory and related diagnostic programs, we will try to establish the validity and applicability of rescaling concept as below.

4. Symptom rescaling in condition monitoring of nonstationary machines

One of the critical activity in deep mining is the ventilation of shafts carried out by continuous running of huge fans, with mass of the rotor of several tones. But due to the unpredictable nature of the mining activity, the demand for the air is unstable, influences strongly the vibration amplitudes measured by vibration condition monitoring subsystem. Beside that, the vibration condition monitoring was implemented in several fans of one Polish cooper mine, as it gives some additional insight into the rotor unbalance as the fault No 1, and the condition of the slide bearings as the fault No 2. On each fan five vibrational symptoms have been measured once a week, but with no information on the motor driving power, or air flow demand and load *l*.

As symptoms of condition, the radial vibration velocity of two bearings, has been taken as all pass quantities, and additionally they were filtered with rotational and blade frequencies, and also overall vibration velocity reading of the fan foundation were used. This gives altogether five symptoms of condition, plus linear life time measure, as the base for creation of symptom observation matrix (**SOM**). These observations were treated by special software (*pcainfo.m*), based on **SVD** as above, and written in *Matlab*® environment. The results of introductory application of this software, without rescaling are shown on Figure 1 containing four pictures.

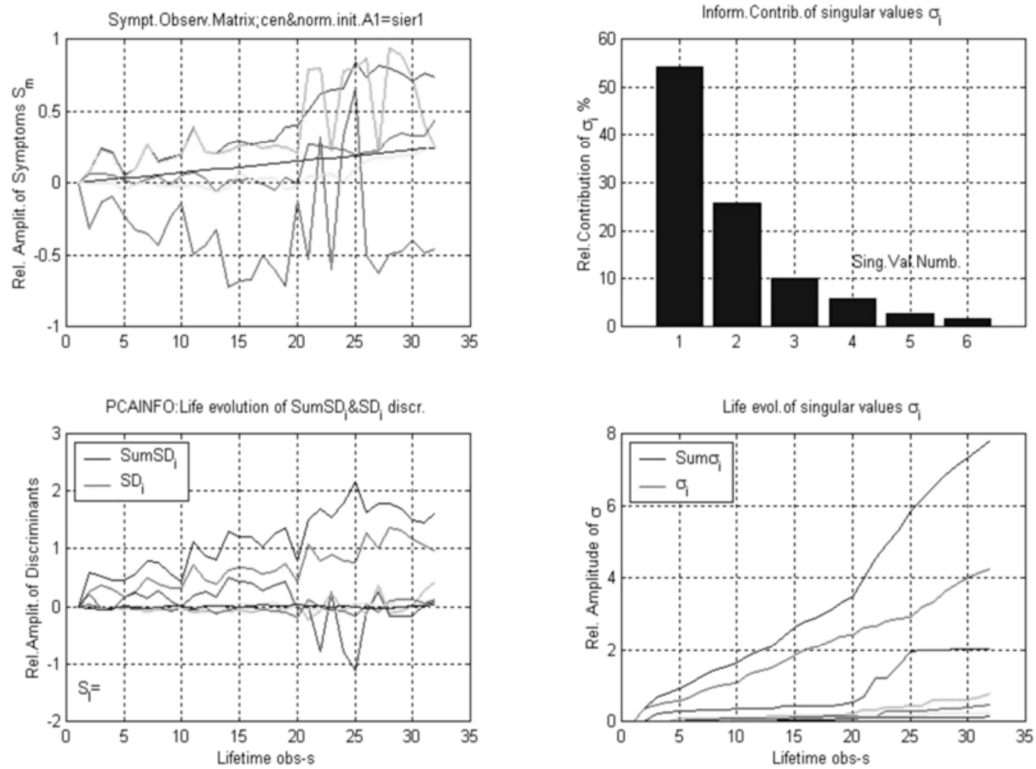

Fig. 1. Multi symptom vibration condition monitoring of a huge fan used to supply air into the deep mining shaft, and the results of ***SVD*** *performed on* ***SOM****.*

As for now, there is no measurement of air flow ***(L)***, or momentary power of electric motors driving the fans. Hence, there is instability in some vibration readings, as it can be seen from picture upper left of Fig. 1, where symptom observation matrix (**SOM**) is graphically presented along **32** weeks life time $\boldsymbol{\theta}$. The upper right picture gives us the percent of information amounts for the extracted independent components ($\boldsymbol{\sigma_I}$) contained in **SOM**, what is equivalent to information distribution of independent generalized faults. The profiles of these faults $\boldsymbol{P_t(\theta)}$, $\boldsymbol{t=1,2,3}$, and the summation generalized symptom $\boldsymbol{SumSD_i}$, calculated according formulae (8) – (10), are shown on the lower left picture of Fig.1.

Comparing now the upper and the lower left pictures of (Fig.1) one can notice that **SVD** decomposition stabilizes the course of generalized symptoms (*left bottom picture)*, because $\boldsymbol{SD_i}$ oscillation due to air flow variability are much smaller than the primary one. From the picture top right one can notice two independent information on evolving faults, probably the rotor unbalance and slide bearing deterioration. Hence, one can conclude, that multidimensional symptom observation can lead us to much better condition assessment than by means of one symptom only. And the best generalized symptom for further condition assessment seems to be the upper curve and the second one on the bottom left picture, what corresponds in our calculation to summation quantity $\boldsymbol{SumSD_i}$, and the first generalized fault $\boldsymbol{SD_1}$.

As there was no rescaling in all calculations of Fig.1, let us assume, that the first observation of our symptom vector (*first row of* ***SOM***) was performed during the high demand of air in the ventilation system, so during $\Delta l > \boldsymbol{0}$. Corresponding rescaling coefficient (2) must

be less than one, and the rest of the observations in **SOM** we leave unchanged. Due to this assumption, our rescaling diagonal matrix will be: ***L_{pp} =Diag(0.9;1;1;....)***.
Following this change in **SOM** let us perform all calculations again by means of similar software with rescaling, namely *pcainfo3.m* The results of such calculations are presented on Fig.2, where one can notice, that every of four pictures has some relevant change. The left upper picture presents much smaller oscillations than previously (Fig.1), there are less negative value symptom (*negative after standardization of column*), and the information amount prescribed to generalized fault No 1 has grown up almost to 60%. One can notice much more on the picture lower left, where almost all oscillations of previously negative generalized symptom has changed to positive valued, and the dynamics of summation generalized symptom ***$SumSD_i$*** and the first one ***SD_1*** , is much larger than before. In a similar way one can observe the increase of dynamics for a symptoms ***$Sum\sigma_i$*** and ***σ_1*** .

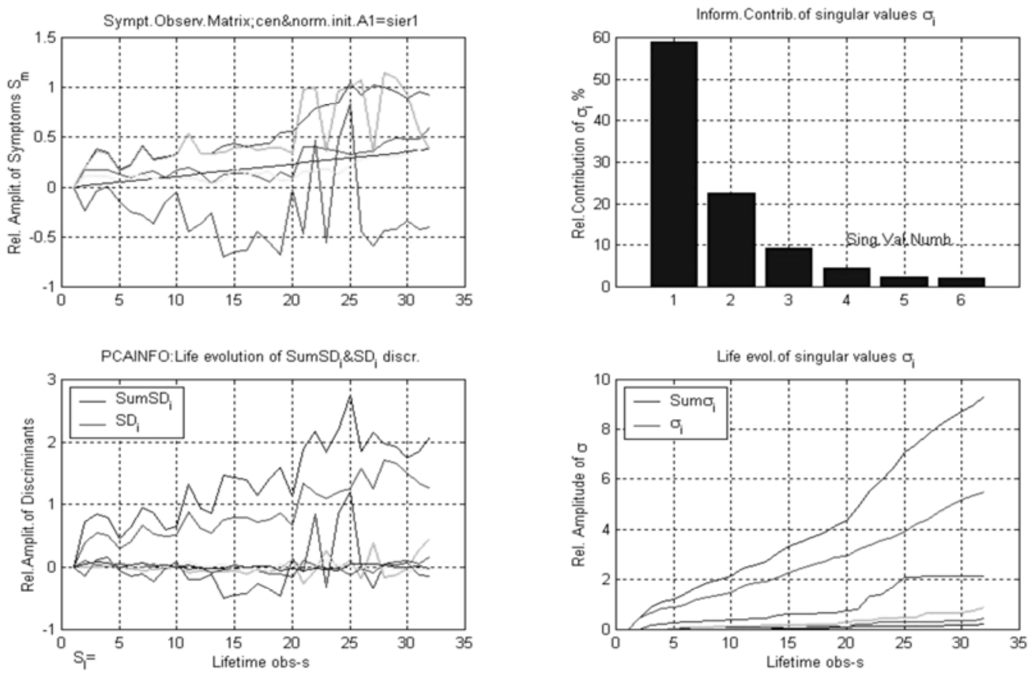

*Fig. 2. Data the same as on Fig 1, but with rescaling of the first raw of **SOM**, with rescaling matrix: **L_{pp}=Diag(0.9;1;1...).***

Considering once more the oscillations of symptom values in case without rescaling (*Fig. 1 upper left*) one can find, that these oscillations have larger amplitudes after observation No 20, having large maximal and minimal values. This may mean that primary cause of this vibration instability was the change of the fan load. Let us take this observation into account and change the respective rescaling coefficients, this time increasing them up to ***l_p = 1.1***. Hence in the next approach we will put this rescaling coefficient into the rows of **SOM** No; 23; 26; 30; 31; 32, together with the l_p = 0.9 for the first observation, as before. The results of a such recalculations with above defined rescaling matrix are shown on Fig. 3.

Comparing now the results of latest calculations (Fig. 2), with unmodified case of Fig. 1, one can notice successive decrease in symptom oscillations, and the increase of the dynamics of generalized symptoms ***$SumSD_i$*** and ***SD_1*** . What is more to notice here, it is increase of information contribution of the main component ***σ_1*** up to 60%, and the increase in the dynamics of generalized symptoms ***$Sum\sigma_i$*** and ***σ_i***, up to the value of **10**. Concluding this positive change in the life course of generalized symptoms one can suppose, that by the rescaling operation of symptom values it is possible to decrease symptom variability, making them more monotonic, like in case of continuous running with the same system load. Also, by the same rescaling operation one can increase the dynamics of the generalized fault symptoms. Maybe the same is true for the generalized symptom life curve.

It is worthwhile to analyze more the influence of rescaling, in particular the influence on the quantities shaping the final maintenance and operational **go / no go** decisions. The key quantity is here symptom limit value S_l calculated from generalized fault symptoms SD_1 or $SumSD_1$. As it was mentioned during introduction, this quantity can be calculated using the concept of symptom reliability ***R(S)*** [9]. This quantity can be calculated continuously by the software of *pcainfo3.m*, and on the Fig. 4 one can find the coarse of symptom limit value S_l for the above shown cases of the fan **sier1**, marked here as **case1** till **case 3**. This means that **case 1** is without rescaling , and **case 2** and **3** concerns respectively Fig. 2 and 3.

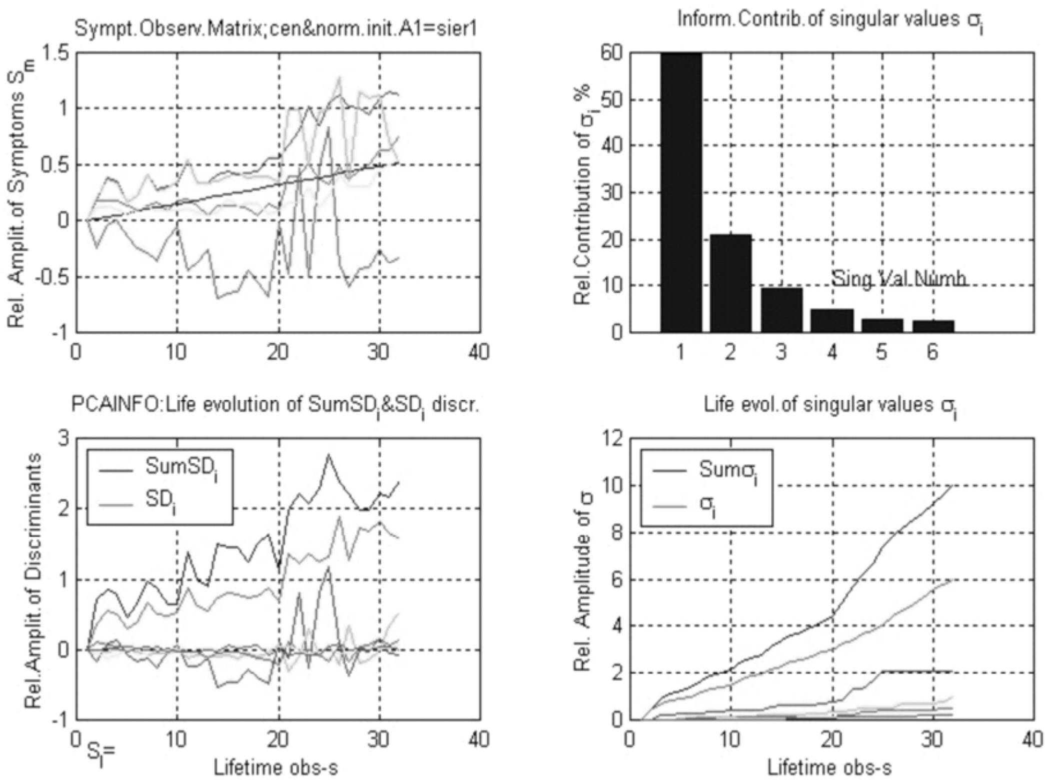

Fig.3. Multidimensional CM of a huge fan as on Fig 1 and 2 but with another rescaling $\boldsymbol{L_{pp}}$*; for the observations No: (1)=0.9; (23)=1.1; (26)=1.1; (30-32)=1.1.*

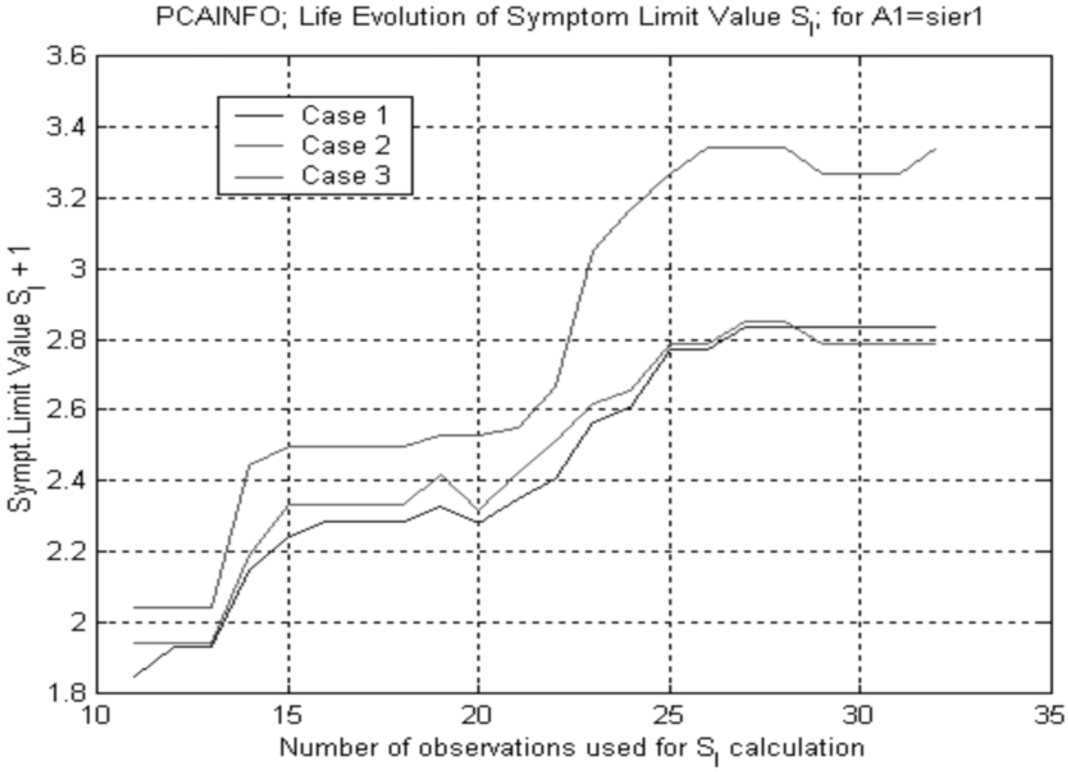

Fig. 4. Rescaling influence on the value and the life coarse of symptom limit value $\boldsymbol{S_l}$ *calculations,*
for the generalized fault symptom of the fan ***sier1*** *according to different rescaling as on Fig.1 (case 1) up to Fig. 3 (case 3).*

One can conclude from the S_l coarse of Fig. 4, that slight rescaling modification as for the case 2 gives almost no change in S_l, but rescaling of several symptom observation as in case 3 give the change of the coarse and the value of the symptom limit value S_l.

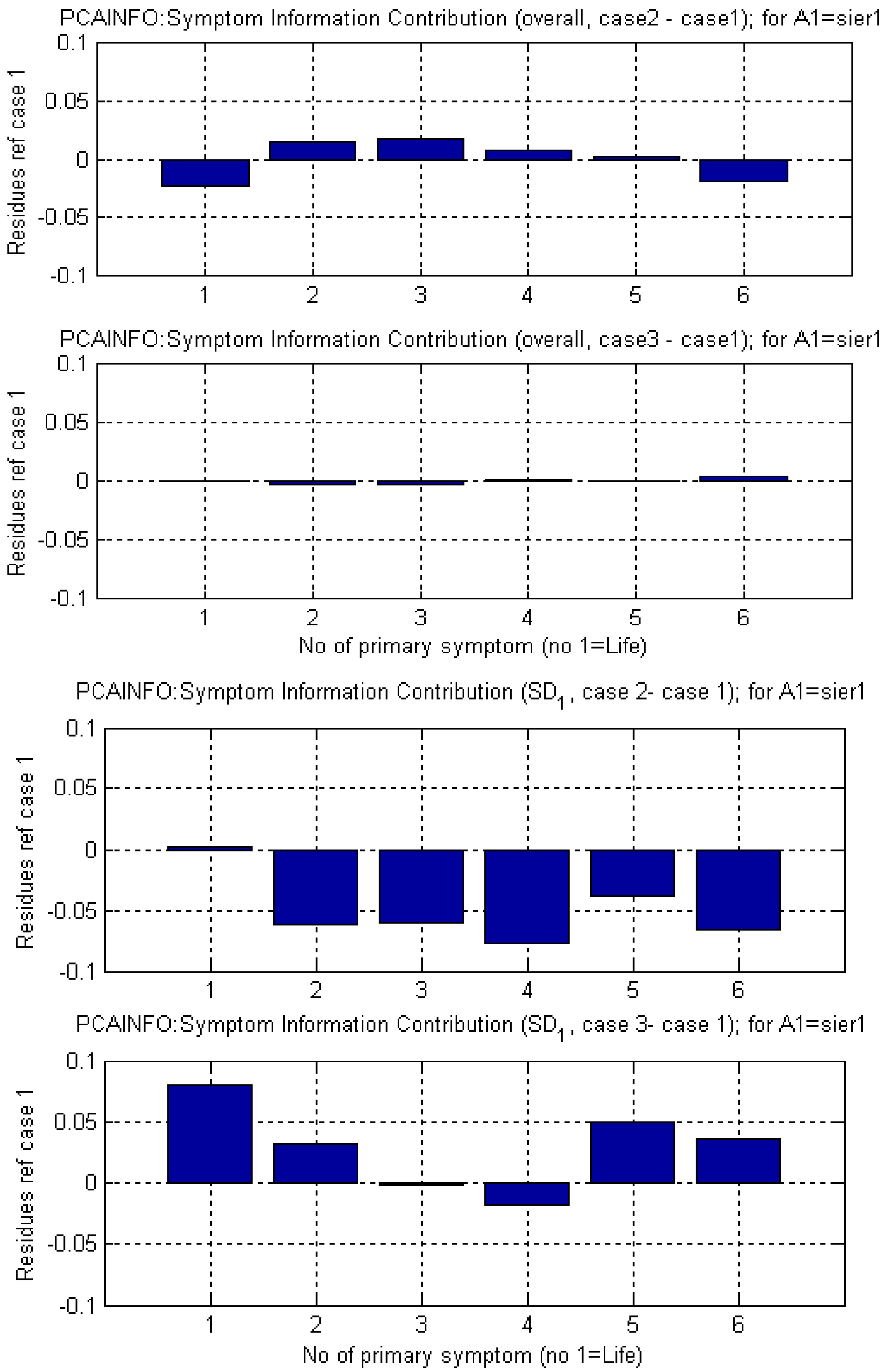

*Fig. 5. Overall and partial (SD_1) sensitivity of information contribution of primary measured symptoms to rescaling operation as in **case 1** to **case 3**.*

The last question in investigating the rescaling influence to vibration condition monitoring, is the sensitivity of this operation to the relative symptom information contribution mainly; overall contribution to the total information resource in **SOM**, and information contribution to the first generalized fault symptom ***SD₁*** . This question is shown on Fig. 5 in a synthetic manner as the differences between**: cases 2 - case 1**, and **case 3 - case 1**. One can see from the fig. 5 upper picture, that the overall information resource of **SOM** do not change almost in response to symptom rescaling, and the same concerns to the first generalized fault symptom $\boldsymbol{SD_1}$, where the maximal difference is 0.075 of its primary information contribution. This small change is quite visible on the lower part of Fig. 5.

This is all what can be said at this introductory stage of the development of this concept concerning the symptom rescaling in multidimensional condition monitoring, and its influence on some decisive quantities. The more can be said when we will have sensor data concerning the change of the machine load and / or environmental conditions described by a logistic vector $\boldsymbol{L}$. But of course the presented idea of symptom rescaling has some limitations and it does not give the cure for all unstable and noisy symptom readings in the practice of vibration condition monitoring.

5. Conclusions

It follows from the above consideration, supported by the real condition monitoring data, that the idea of rescaling of **SOM** in multidimensional condition monitoring seems to be sound and practical in application to some real critical machine with unstable loading. It was shown, that primary unstable symptoms with much oscillations are more stable after application of **SVD,** and much more monotonic with greater dynamics, when rescaling of some observation was in use. Also, the influence of rescaling to the assessment of symptom limit value $\boldsymbol{S_l}$ is not a large. Hence, having some **sensor of machine load** in our condition monitoring subsystem, we can use it and learn iteratively how to calculate rescaling coefficients for every case of symptom observation, in a non standard load or environment condition. Next problem we should solve here is the definition of the main criterion for rescaling; it may be not only the stable life course of generalized fault symptoms $\boldsymbol{SD_1}$, as it was above. We can use also as a criterion the stable value of symptom limit value $\boldsymbol{S_l}$**,** or the emergence of next generalized fault $\boldsymbol{SD_2}$, and the other quantity particular to the given object. It seems to be worthwhile to consider these questions designing new implementation of condition monitoring subsystem to some critical machinery. But this must be done separately for the given machine type and the case of implementation.

6. References

1. Cempel C., Innovative developments in systems condition monitoring, Keynote Lecture, Proceedings of **DAMAS'99** Conference, Dublin 1999.
2. Natke H. G., Cempel C., **Model Aided Diagnosis of Mechanical Systems**, Springer Verlag, Berlin - Heidelberg, 1997,p248.
3. Cempel C., Multidimensional Condition Monitoring of Mechanical Systems in Operation, **Mechanical Systems and Signal Processing,** 2003, **17**(6), 1291 – 1303.
4. Cempel C, Implementing Multidimensional Inference Capability in Vibration Condition Monitoring, Proceedings of **Acoustical and Vibratory Surveillance,** Senlis - France, October 2004.
5. Gere G. M., Weawer W., **Matrix Algebra for Engineers**, Van Nostrand, Princetown, 1965.
6. Natke H. G., Cempel C., The symptom observation matrix for monitoring and diagnosis, **Journal of Sound and Vibration, 248,** 597 – 620, 2002.
7. Kiełbasiński A., Schwietlick H., **Numeryczna Algebra Liniowa**, WNT, Warszawa, 1992, (***Numeric Linear Algebra**, in Polish*).
8. Golub G. H., VanLoan C. F., **Matrix Computation**, Oxford, North Oxford Academic, 1983.
9. Cempel C., Natke H. G., Yao J. P. T., Symptom Reliability and Hazard for Systems Condition Monitoring, **Mechanical Systems and Signal Processing**, Vol. 14, No 3, 2000, pp 495-505.
10. Will T., Hanger matrix, Two-thirds Theorem, **Internet;**

http://www.uwlax.edu/faculty/will/svd/svd/index.html ,2005; (*see also there: SVD ingredients Mathematica, April 2004*)
11. Cempel C., Tabaszewski M., Multidimensional vibration condition monitoring of nonstationary systems in operation. Proceedings of **International Congress on Sound and Vibration,** (*on CD-paper No 496*), Lisbon, July 2005.
12. Tumer I. Y., Huff E. M., Principal component analysis of tri-axial vibration data from helicopter transmission, Proceedings of **56th Meeting of the Society of Machine Failure Prevention Technology,** Virginia Beach, 2002.
13. Pantopian N. H., Larsen J., Unsupervised condition detection in large diesel engines, **Proceedings of the IEEE Workshop on Neural Networks**, 1999.

Figure Captions

Fig. 1. Multi symptom vibration condition monitoring of a huge fan used to supply air into the deep mining shaft, and the results of ***SVD*** *performed on* ***SOM****.*
Fig. 2. Data the same as on Fig 1, but with rescaling of the first raw of ***SOM****, with rescaling matrix:* $\boldsymbol{L_{pp}=Diag(0.9;1;1...)}$.
Fig.3. Multidimensional CM of a huge fan as on Fig 1 and 2 but with another rescaling $\boldsymbol{L_{pp}}$*; for the observations No: (1)=0.9; (23)=1.1; (26)=1.1; (30-32)=1.1.*
Fig. 4. Rescaling influence on the value and the life coarse of symptom limit value $\boldsymbol{S_l}$ *calculations,*
for the generalized fault symptom of the fan **sier1** *according to different rescaling as on Fig.1 (case 1) up to Fig. 3 (case 3).*
Fig. 5. Overall and partial (SD_1) *sensitivity of information contribution of primary measured symptoms to rescaling operation as in* ***case 1*** *to* ***case 3****.*