FAULT ORIENTED DECOMPOSITION OF SYMPTOM OBSERVATION MATRIX FOR SYSTEMS CONDITION MONITORING
- Case Study -
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Contents

Abstract

In critical system condition monitoring we observe several symptoms of condition, mostly of vibrational nature, and by discrete way of symptom observation so called symptom observation matrix is created. The real problem is now how to extract from this matrix all condition related information, and to distinguish different system faults evolving during its operation. The method proposed in this report depends on application of singular value decomposition (SVD) and special transformation to the symptom observation matrix. Due to this new condition related measures and indices were created and applied to specially chosen test symptoms, and to symptom observation matrix of real systems in operation as well. It was found that, application of SVD to systems condition monitoring can bring some progress by reducing symptom redundancy, distinguishing faults, and helping in the area of condition inference, when applying new SVD related condition and fault indices.

1 Introduction

The goal of system condition monitoring depends on the assessment of its residual life, (see for example [2], [1], [6]), and action to be taken for further prolonged, and safe operation of a critical system. The assessment itself, depend in turn on the solution of inverse illposed problem, like below. We have a set of symptoms [1] observed during the continuous system operation. We know that during system operation its physical and operational condition (characteristics) deteriorates, and we can assess it in terms of damage, faults, defined as a system and relative model modifications. I the best case of critical systems condition monitoring, we describe the condition of the system in terms of life dependent faults $F_t(\theta), t = 1(1)w$. We try also to assume in a first approach, that they are not linearly dependent, like for example the corrosion and the fatigue. Of course these assumptions are mission dependent for a given system, and its validity must be checked, in each case separately. We will not touch here the symptom based inference - fundamental problem of condition monitoring, as for example user can calculate and use the symptom based reliability, (see for example [12], [14]) for that purposes. We will not discuss here very important problem, how to chose the set of proper symptoms, as we have described it somewhere [2], and lately particular in [6]. Our attention will be focused instead, on the fault dependent meaning of observed symptoms evolution during the system operation.

The **Symptom Observation Matrix** (SO-matrix) $O_{pr} \in \mathbb{R}^{p \times r}$ is assembled from measured symptoms: p, the row dimesnsion, is the number of observations per symptom and r, the number of columns, is the number of observed symptoms. Our goal is to extract condition dependent characteristics from the SO-matrix O_{pr}, and to interpret them in terms of system wear, faults, and other operational characteristics, i.e. system modifications. Thus, for a critical system[2]in operation we will pass from *one symptom based inference* to the *SO-matrix based inference.*

For the scope of this paper these problems to be solved can be formulated as below.

1. What are the damage or fault indices, or measures, possible to extract from O_{pr}.

2. What is the evolution and the advancement (or intensity) of system defect (modification), which can be described by some generalized symptom S, possible to obtain from the symptom observation matrix O_{pr}.

3. What types of system physical faults, or generalized faults $F_t(\theta)$, can be obtained from O_{pr}. Here $F_t(\theta)$ describes the type and the life evolution of fault 't', which can be physically well defined in a given case, or can be only some composite characteristics of wear type like, linear, cubic, asymptotic, type of wear, etc.[3]

[1]A *Symptom* is a measurable quantity covariable with system condition.

[2]A system is critical if its malfunction can cause severe loses in terms of human life or large money.

[3]Wear distinction ability seems to be system, or even mission dependent.

4. What kind of transformation of symptom observation matrix O_{pr} we should apply, to obtain above specified indices and characteristics of system modification.

This paper brings some new results obtained by a model investigation and also from the domain of real systems in operation as well, being in line of just enumerated goals. This was possible by using the singular value decomposition (SVD), [15], [16], of symptom observation matrix already proposed in [5]. Some possible interpretation of symptoms life behaviour are given as well as some new damage indices, and measures, flowing from SVD of symptom observation matrix - O_{pr}. But in general the problem is new, and lacks many practical application and mission dependent validation.

2 Systems in Operation and their Condition Observation

Having specified a critical system in operation, and its operational characteristics, like speed, load, etc, we have possibility to assess the system breakdown time - θ_b, at least by reliability investigation [8]. Hence we can write the life dependence of our observed symptoms $S(\theta)$ in terms of life θ, and at the same time some simplified damage measure [2]

$$D = \frac{\theta}{\theta_b}, \quad 0 \leq D < 1. \tag{1}$$

Here θ is the current system life in weeks, months, etc, and θ_b the anticipated system breakdown time measured in the same units.
Thus we have possibility to write down the observed set of symptom in dependence of damage D as below

$$S_m(\theta) = S_m(\frac{\theta}{\theta_b} \cdot \theta_b) = S_m(D \cdot \theta_b), \quad m = 1(1)r. \tag{2}$$

Usually we are making observations of symptom over some life distance $\Delta\theta$, in which the condition of operating system, so the values of observed symptoms, may change substantially. In this way condition monitoring depends on 30 to 50 (or more depending on the stability of system load) symptom readings taken over the life span of a system $0 \div \theta_b$. Hence the particular reading of symptom $S_m(\theta)$ at the time: $\theta_n = n\Delta\theta < \theta_b$ will be

$$S_m(\theta_n) = S_m(n\Delta\theta) = S_{nm}, \quad m = 0(1)r, \quad n = 0(1)p. \tag{3}$$

If we measure now some number of symptoms, say r, over system life $0 \div \theta_p = p\Delta\theta$, we can create already mentioned symptom observation matrix: O_{pr}, [5], [6], in the form

$$O_{pr} = [S_{nm}]; \quad n = 0(1)p, \quad m = 0(1)r. \tag{4}$$

Observed symptoms can have different physical origin, like temperature, vibration, or even different signal processing during the measurement, like in case of frequency filtering of vibration process of some part of mechanical system. Hence symptoms will have different physical units, range, and different initial values at the beginning of system monitoring $\theta_o = 0$. Due to that and intended common processing of matrix O_{pr}, every symptom reading is recommended [6] to be centered and normalized columnwise.
One can anticipate, that different methods of centering by some reference value, and normalization - by some other reference or statistical value, may amplify or diminish, some fault information contained in O_{pr}, which we are looking for, (see items 1 - 4 of introduction), because this matrix transformations are not orthogonal.
For the centering of symptom readings we have various possibilities; to subtract symptom initial value $S_m(0) = S_{om}$, column average value, or a pre - chosen reference value. The centering with respect symptom initial value S_{om} give us the invariance with respect of starting condition of the

system, but other possibilities must be investigated too.
For column (symptom) normalization we have also various possibilities; to normalize it by symptom initial value S_{om}, - constant over the system life, and to normalize by column average value, or root mean square (rms) value. But we should remember, that average and rms values will change with the growing number of observations.
Summing up one can say, that each transformation of symptom observation matrix O_{pr}, either centering or normalization is not orthogonal, and can give us different possibilities of inference on system condition, i. e. faults, modes of wearing or modifications. Hence we will investigate them in the next sections, but first let us remind the theory of SVD in application to symptom observation matrix [5], [6].

3 Singular Value Decomposition (SVD) of the Symptom Observation Matrix

The idea of SVD applied to a symptom observation matrix O_{pr} can be presented shortly as below [15], [16].

$$O_{pr} = U_{pp} * \Sigma_{pr} * V_{rr}^T, \;\; T - transposition, \tag{5}$$

with U_{pp} and V_{rr} unitary left hand and right hand singular vector matrices of respective order, and subdiagonal singular value matrix

$$\Sigma_{pr} = diag(\sigma_1, ...\sigma_l), with\ \sigma_1 \geq \sigma_2 \geq ...\sigma_z > 0, \tag{6}$$
$$\sigma_{r+1} = ...\sigma_l = 0, \;\; l = max(p, r).$$

Another form of decomposition, much convenient to us is given below in the second line [5]

$$O_{pr} = U_{pp} * \Sigma_{pr} * V_{rr}^T \tag{7}$$
$$= \Sigma_{t=1}^z \sigma_t \cdot (u_t * v_t^T) = \Sigma_{t=1}^z (O_{pr})_t.$$

with the singular values σ_t and u_t, v_t - singular vectors (orthogonal) - as columns of respective matrices. Please note they are creating together submatrices $(O_{pr})_t$ of SVD, describing totally the given mode of system modification (wear).
In this way, by SVD, the information resources of O_{pr} are decomposed into several independent sources described by $\sigma_t, u_t, v_t, and : (O_{pr})_t$, and we can trace the evolution of these quantities in the system life time θ. Using singular vectors u_t, v_t we can get the interpretation of system condition evolution in terms of so called generalized symptoms [5]

$$\mathrm{SD}_t = O_{pr} * v_t = \sigma_t \cdot u_t. \tag{8}$$

Another type of generalized symptom or fault indicator, with much greater range of evolution can be obtained from singular submatrix $(O_{pr})_t$ by summation over its columns [5]. Defining

$$\mathbb{I}_r := (1, \ldots, 1)^T \in \mathbb{R}^r, \tag{9}$$

to be the vector of length r whose compoents are all one we have

$$\mathrm{SG}_t := (O_{pr})_t \mathbb{I}_r = \sigma_t u_t v_t^T \mathbb{I}_r \tag{10}$$
$$= \sigma_t \left(\sum_{i=1}^{p} v_{t,i} \right) u_t \tag{11}$$

Equations 8 and 11 show that SD_t and SG_t as scalar multiples of the vector u_t correlate perfectly.
Using SVD results we can also create the combined measure and indices from symptom observation

matrix. Calculating determinant of economic decomposition, as product of nonzero singular values we have

$$\mathrm{PsDet}(O_{pr}) = D1 = \prod_{t=1}^{z} \sigma_t > 0, \quad t = 1(1)z. \tag{12}$$

Knowing singular values of symptom observation matrix allow us to calculate the value of its Frobenius norm [15], and as operation of SVD is orthogonal transformation of O_{pr} it is than obvious that: $\|O_{pr}\|_F = \|\Sigma_{pr}\|_F$.

$$\|O_{pr}\|_F = \sqrt[2]{\Sigma_{i=1}^{z} \sigma_i^2} = AF \tag{13}$$

Squaring of singular values as above gives preference to the bigest one,(the first singular value), and the rest of them gives samall influence to the value of Frobenius norm. Instead we can use the first order norm of subdiagonal singular value matrix Σ of the observation matrix O_{pr}, in the form:

$$AS = \sum_{i=1}^{z} |\sigma_i|,$$

or as its approximation, the first order norm of transformed symptom observation matrix.
Having such fault sensitive and compoosite measures of system modification as above, we can trace them as a function of system life θ . They will give us different information concerning the evolution of damage;
σ_t will show the evolution of some generalized fault, and $D1(\theta), AF(\theta)$, $AS(\theta)$ will present to us the damage (condition) of system in general, and in a some composite way.
Going from these properties in line with our introduction to the paper, we will introduce now the fault evolution functions $F_t(\theta)$, and basing on them one can make following supposition.
Let $F_t(\theta)$ will be covariant with σ_t, i. e.

$$F_t(\theta) \sim \sigma_t(\theta), \tag{14}$$

what gives us immediately

$$D1(\theta) = \prod_{t=1}^{z} \sigma_t \sim \prod_{t=1}^{z} F_t(\theta) = CF(\theta) \tag{15}$$

$$AF(\theta) = \sqrt[2]{\Sigma_{i=1}^{z} \sigma_i^2(\theta)} = AF(\theta).$$

This damage indicators we may call cummulative generalized fault measures $CF(\theta)$, $AF(\theta)$, $AS(\theta)$, and it will be interesting to trace their evolution, depending both on system life as well as symptom observation matrix transformation.

4 Transformation from Symptom Space to Fault Space

The symptom observation matrix O_{pr} generates by its different r columns the symptom space. But in many cases symptoms depend on each other, or depend commonly on other not measurable directly life process, like corrosion, etc. So, the symptom columns are not orthogonal. What we want for systems condition monitoring is to transform the symptom space into the fault space, the space in which wear or system condition modification proceed. This space will allow us to observe the system life or operation in terms of fault evolution $F_t(\theta)$, $t = 1(1)z$. In the best case of such transformation to fault space one may obtain $F_t(\theta)$ being some measures or indices describing the evolution of real system damage process during its life. In some other mission dependent cases, our fault measure $F_t(\theta)$

will describe the essential form of wear process or system properties modifications, like the linear, cubic, quadratic growth, or the like. In [5] it was shown, that application of SVD to the original and untransformed symptom observation matrix O_{pr}, provides certain fault indices and measures, SD and SG, covariable with the system condition. At the same time, these indices have much higher dynamics of evolution than the originally measured symptoms $S(\theta)_r$. Hence by using them it was much easier to determine the system condition, and its residual life. It was also postulated in [5], that the largest singular values, e.g. $\sigma_1, and \sigma_2$, are measures of wear advancement in an operating critical system. The trouble in SVD application is, that there may be several physical modes of system condition deterioration, which will give similar change in our generalized fault $F_t(\theta)$, and we will not distinguish them. Also of great importance is to be able to distinguish the static behaviour, i.e. no change in the system condition, from the system condition evolution. This is not possible by perfoming the SVD-analysis of the original symptom observation matrix since in the case of a static system behaviour we have exactly one singular value coming from the constant/static and usually nonzero observation columns (rank=1) of the original observation matrix.
To define the above motivated transformations in detail we recall that each column $o^j \in \mathbb{R}^p, j = 1, \ldots, r$, of the symptom observation matrix $O_{pr} \in \mathbb{R}^{p \times r}$ is a fixed-step time series of observations/measurements of certain **s**ymptoms $s^j \in \mathbb{R}, j = 1, \ldots, r$, of a system, e.g. corrosions, modal data, strains, crack lengthes etc. For each symptom a reference value h^j is asummed to be known representing the **h**ealthy state of the system concerning the symptom $j, j = 1, \ldots, r$. The columns $\tilde{o}^j \in \mathbb{R}^p, j = 1, \ldots, r$, of the Relative-Fault-Observation-Matrix (RFO-matrix) $\tilde{O}$ are affine transformations of the columns of the observation matrix O in the following way:

$$\tilde{o}^j := \left(\frac{o_{1j} - h^j}{h^j}, \ldots, \frac{o_{mj} - h^j}{h^j} \right)^T, \quad j = 1, \ldots, n. \tag{16}$$

From the definition we see that $\tilde{o}^j$ measures the relative deviation of the symptoms from their healthy state. Thus, the singular values and vectors performed by the singular value decomposition (SVD) of the RFO-matrix are indicators for intensities of generalized symptoms/faults, i.e. they are generalized deviations from the healty state of the system represented by the healthy state vector $h = (h^1, \ldots, h^r)^T$. This method is also known in statistics as *Main-Component-Analysis* ([4]) where it is applied to multi-property objects with gaussian distribution. In this case the deviations are measured relative to the corresponding mean values of the gaussian variables. Mathematically it is related to the main axis transformation since the singular values and vectors correspond to the square roots of the eigenvalues and to the eigenvectors of the positive-semidefinite correlation matrices $O^T O$ and OO^T ([3]).
A summary of transformations that will be discussed in the following is:

Relative Fault or Centering and Normalization w.r.t. the healthy state

$$\tilde{o}^j := \left(\frac{o_{1j} - h^j}{h^j}, \ldots, \frac{o_{pj} - h^j}{h^j} \right)^T, \quad j = 1, \ldots, r. \tag{17}$$

Absolute Fault / Centering w.r.t. the healthy state

$$\tilde{o}^j := \left(o_{1j} - h^j, \ldots, o_{pj} - h^j \right)^T, \quad j = 1, \ldots, r. \tag{18}$$

RMS-Transformation / Centering w.r.t. healthy state and RMS-Normalization

$$\tilde{o}^j := \left(\frac{o_{1j} - h^j}{\mathrm{RMS}(o^j)}, \ldots, \frac{o_{pj} - h^j}{\mathrm{RMS}(o^j)} \right)^T, \quad j = 1, \ldots, r. \tag{19}$$

where

$$\mathrm{RMS}(o^j) := \frac{\|o^j\|}{\sqrt[2]{p}} \tag{20}$$

Average-Transformation / Centering w.r.t. Mean-Value

$$\tilde{o}^j := \left(o_{1j} - \text{Mean}(o^j), \ldots, o_{pj} - \text{Mean}(o^j)\right)^T, \quad j = 1, \ldots, r. \tag{21}$$

No Normalization

$$\tilde{o}^j := \left(o_{1j}, \ldots, o_{pj}\right)^T, \quad j = 1, \ldots, r. \tag{22}$$

We begin our considerations looking first to distinguish the static and the evolving system condition. Assuming that nothing is changing, and observing 5 such test symptoms, we will measure them as being parallel to life axis, and after SVD they will give one singular value $\sigma_1(\theta)$ growing linearly with the life θ progress. But applying the centering operation as defined in (18) to the initial symptom value (=*ABSOLUTE FAULT*), as the first transformation of original O_{pr}, we find the absolute fault of O_{pr} being identical to zero, which of course implies that the singular values are all zero. Hence the absolute faulte or centering transformation to symptom initial value (healthy state) allows us to distinguish the static system behaviour, and at the same time to substract the initial (starting) condition of the system.
When one of the above five symptoms begin to express the linear growth, as it is shown on the figure 1, where symptom observation matrix is without symptoms centering (original, (22)), we will obtain the second singular value, and two nonzero generalized fault symptoms $SG1(\theta)$ and $SG2(\theta)$. This case maybe also good illustration of primary poor choice of symptoms, with only one sensitive to the condition change. It is seen also from the two bottom pictures of the figure 1, that it is hard to decide which behaviour; static or linear change, influences mostly the values of σ_1 and σ_2, (pictures top right of figure 1). The rest of figure 1 is self explaining, and PsDet = is the value of determinant as the product of nonzero singular values (see eq 12 and [5]). This composite results of SVD maybe one of the damage indices, as it symbolizes the volume of the fault space, after SVD from the symptom space.
When we perform now the centering to the initial value of the previously shown symptom observation matrix, (picture top left of figure 1), we obtain a very clear situation as shown on figure 2, with only one nonzero singular value σ_1, and one generalized symptom $SG1(\theta)$ describing the linear growth. It is worth to mention too, that even several symptoms of linear growth of different speed will give the same effect, only one singular value, after centering of symptom observation matrix. Figure 2 also shows that life evolution of first generalized symptom $SG1(\theta)$ as the damage index is linear function of system life. It may be of interest to see how the respective singular value σ_1, and the value of determinant $\text{PsDet}(\theta) = D1(\theta)$, and Frobenius norm of symptom observation matrix will change in the situation of one symptom linear growth. This is illustrated in figure 3, where the top left pictures are repeated for comparison of figure 2, and two bottom pictures and one top right give needed illustration of $\sigma_1(\theta), D1(\theta), AF(\theta)$ evolution in this case. It is seen there, that in response to the linear symptom growth the life evolution of new damage indices is nonlinear, growing much more than the original symptom, above the value of 40 in this case. So in real cases of system condition monitoring it may be good to look for the evolution of these new SVD defined damage indices.
Analyzing once more three just shown Figures rises new question concerning the singular value, and our new indices representation, in case of much more complex change of symptom behaviour. Figure 4 presents new symptom observation matrix of five test symptoms; two with linear growth of different speed, one quadratic, one cubic, and one linear with jump increasing ten times the current symptom value. So together with jump in symptom value we have four different modes of symptom evolution. Figure 4 shows this case, with the original not transformed symptom observation matrix. Really, we have here four singular values, smallest not seen on the top right pictures due to scale effect, with quite high determinant D1 value, and not visible contribution of linear symptoms 1 and 5 at the bottom left picture. The highest contribution here to the first generalized symptom gives the jump effect in symptom No 4, and the cubic symptom too. If we transform now the symptom observation matrix by centering to initial value of each column we obtain almost the same situation as it is given on figure 5 with slightly smaller inter-correlation of the first and second generalized fault symptoms.

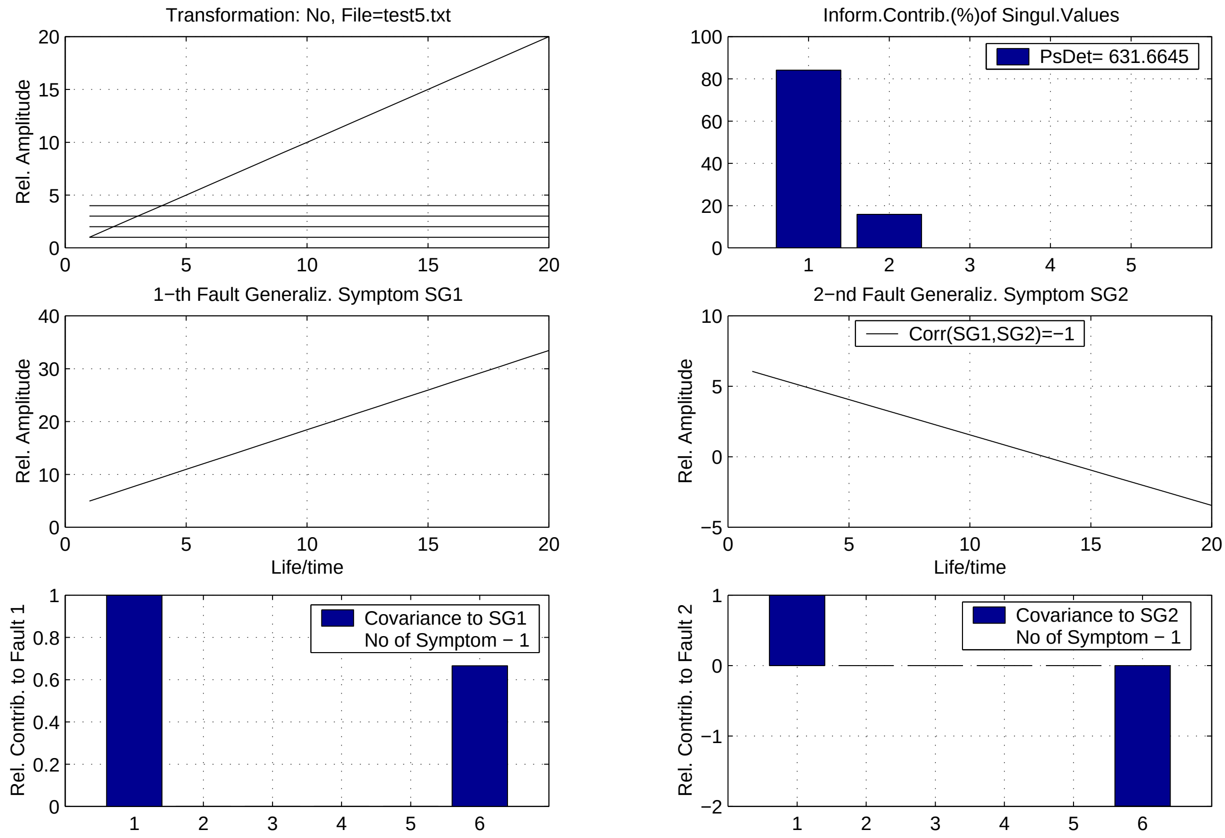

Figure 1: Five not well chosen symptoms, only one sensitive, without centering of symptom observation matrix (original)

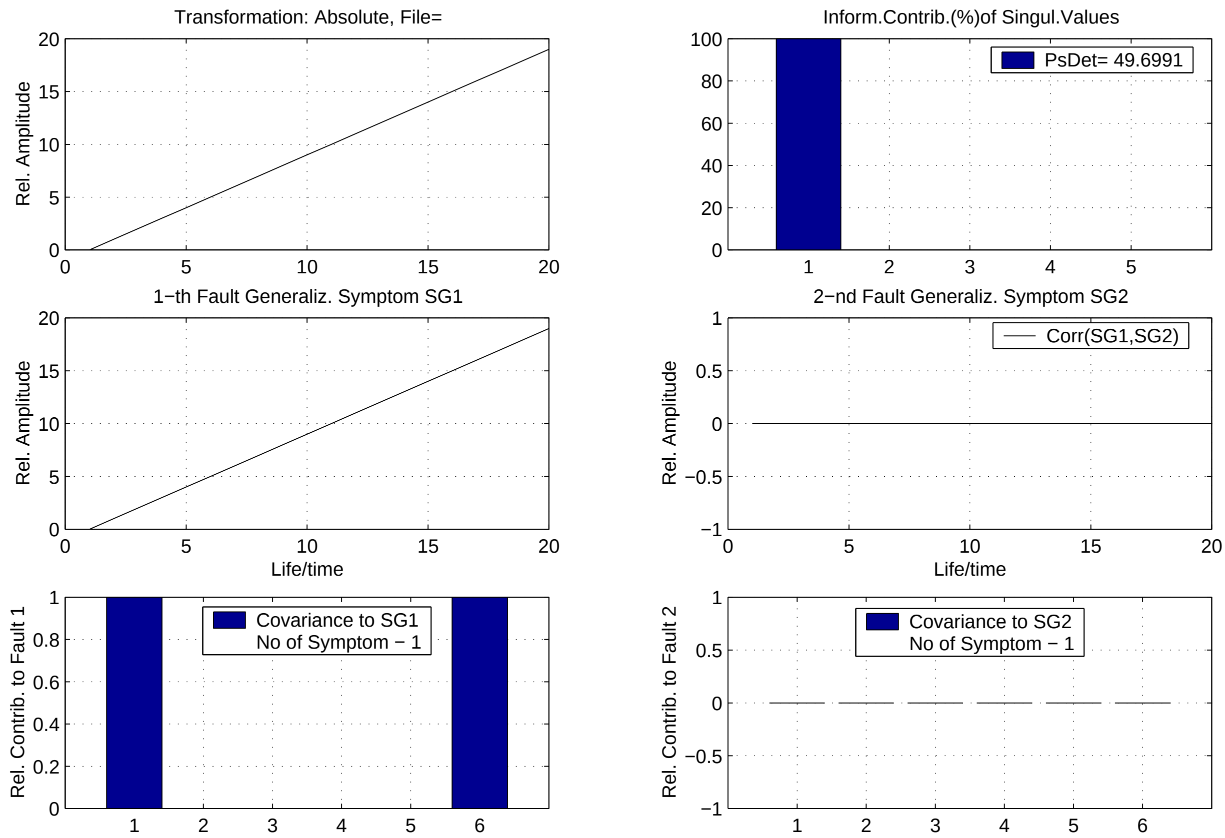

Figure 2: The same situation as on figure 1 but symptom observation matrix initially centered

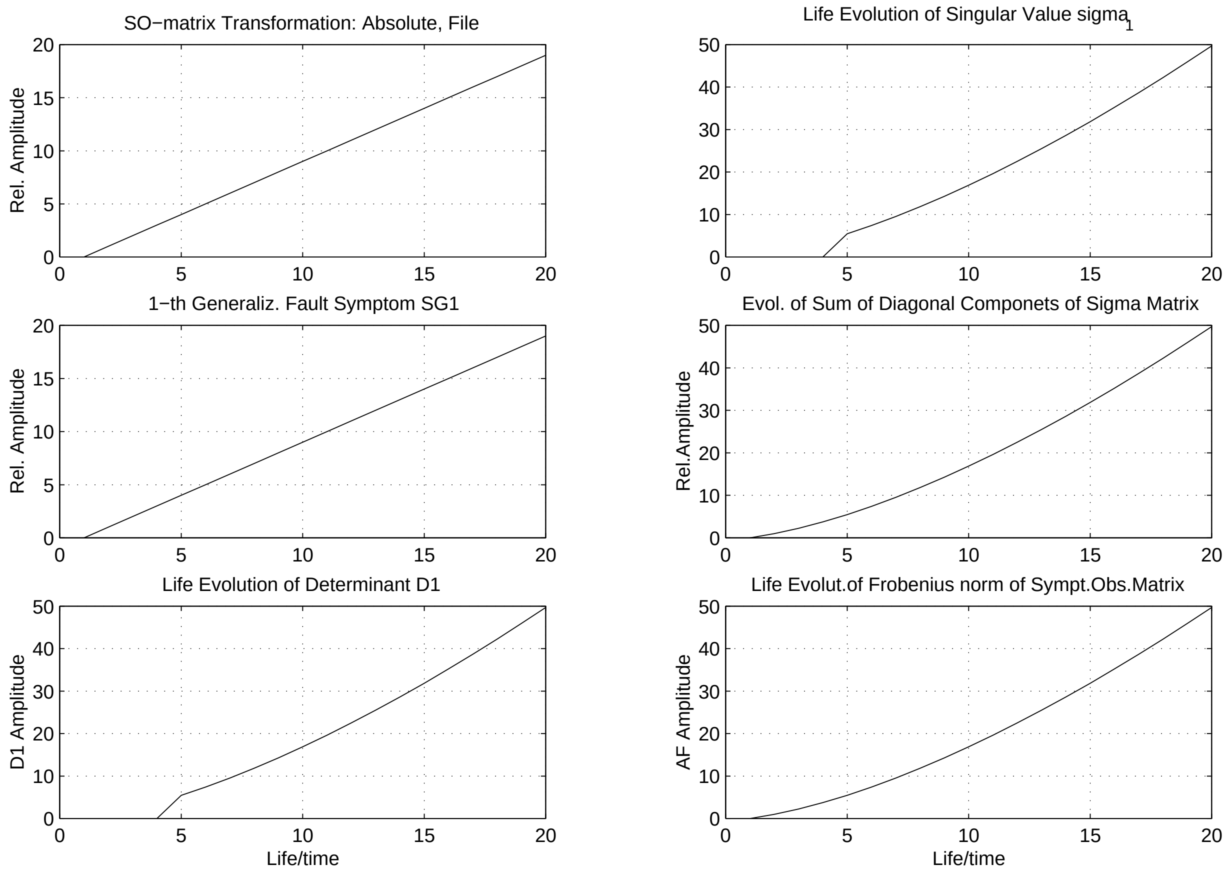

Figure 3: The life evolution of first singular value, determinant value and Frobenius norm, for the same situation as on figure 2

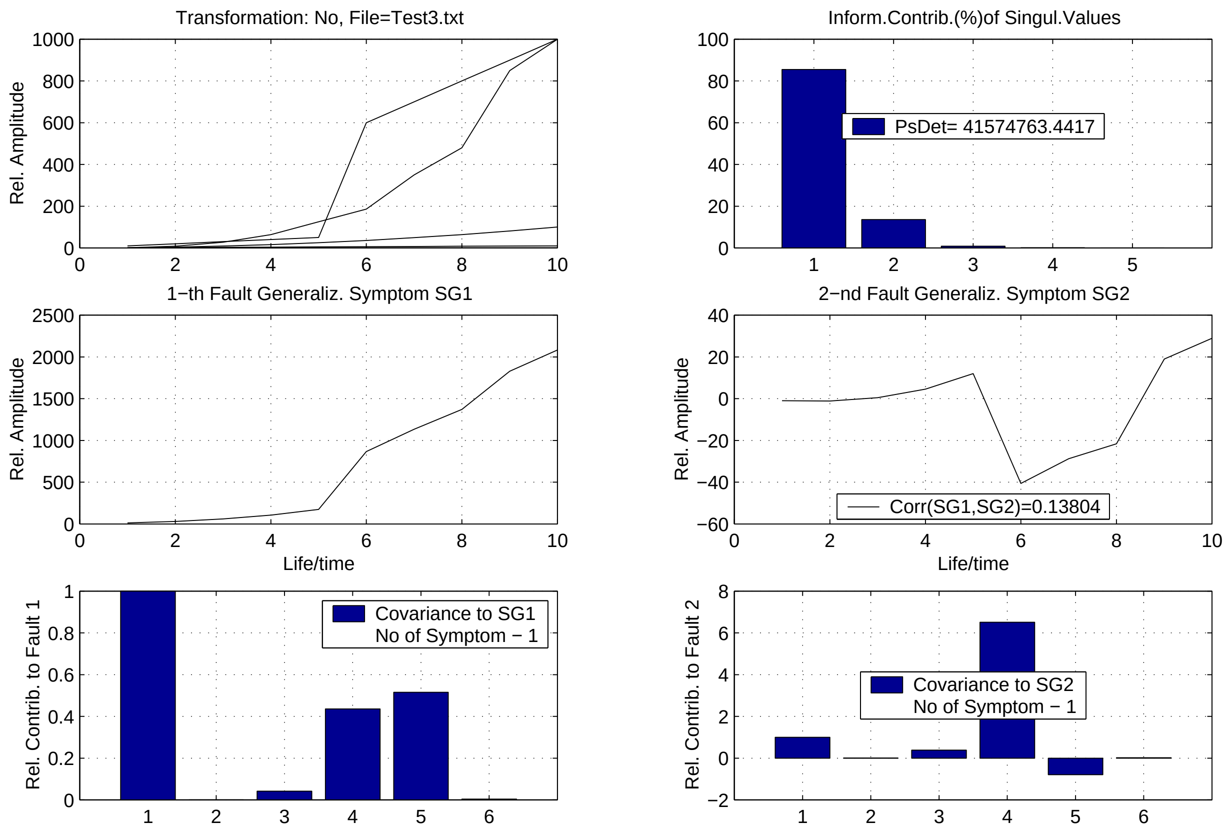

Figure 4: Five symptoms of different original scale with four types of life behaviour, and their SVD representation

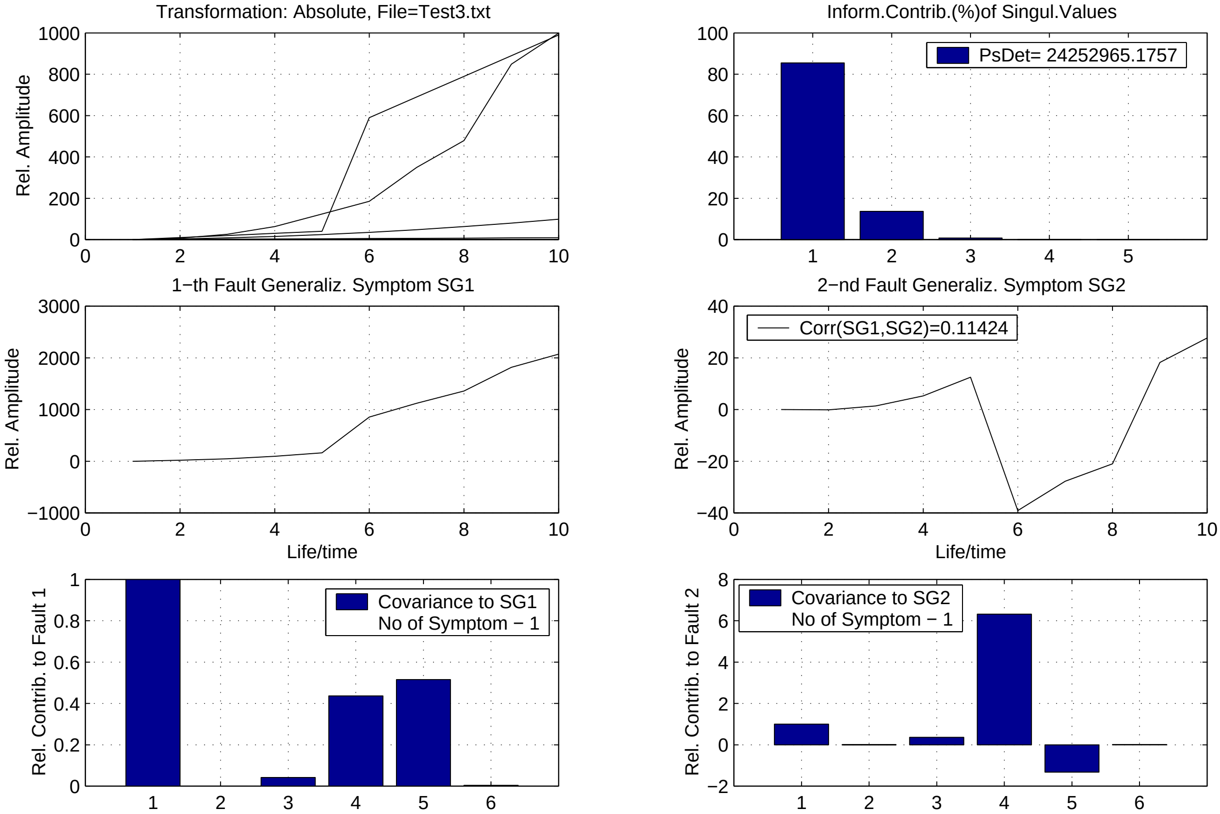

Figure 5: Five symptoms as on figure 4, but after transformation of symptom observation matrix by centering to its initial value

Looking for another possibility of transformation of symptom observation matrix, which allow us to see all nonzero singular values, one can see on figure 6 the case of centering with respect of symptom initial value, and column normalization according to its rms value as defined by equation 19. Really, here all 4 singular values are visible on the top right picture of figure 6, and each symptom gives almost equal contribution to the first and second generalized symptoms SG1 and SG2 - bottom pictures of figure 6. From that point of view, it seems to the best transformation of symptom observation matrix, as all informational resources of O_{pr} are in use. The trouble is that the rms column value is calculated for the current observation number which changes constantly. Also the values of all indices on figure 6 are much lower than in the original case.
Due to these both inconsistencies we will propose to use the normalization with respect of initial value, i.e the relative fault transformation as defined in (17). This situation is shown in figure 7, where the determinant value is much higher and symptom No 3 with cubic behaviour gives the highest contribution to the generalized fault symptoms SG1 and SG2, with much lower inter correlation between them.

Altogether, outside of non transformed symptom observation matrix called here 'original', four other transformation of O_{pr} were calculated and analyzed, namely:

- absolute fault or centering w.r.t. healty state as defined in eq. 18.
- centering by the average value as defined in eq. 21.
- relative fault or centering and normalization w.r.t. healthy state (*Relative Fault*, eq. 17).
- centering by initial value and normalization by column RMS value as defined in eq. 19.

In considering these transformations of symptom observation matrix, not orthogonal in general, the following important resultant characteristics were taken into account.

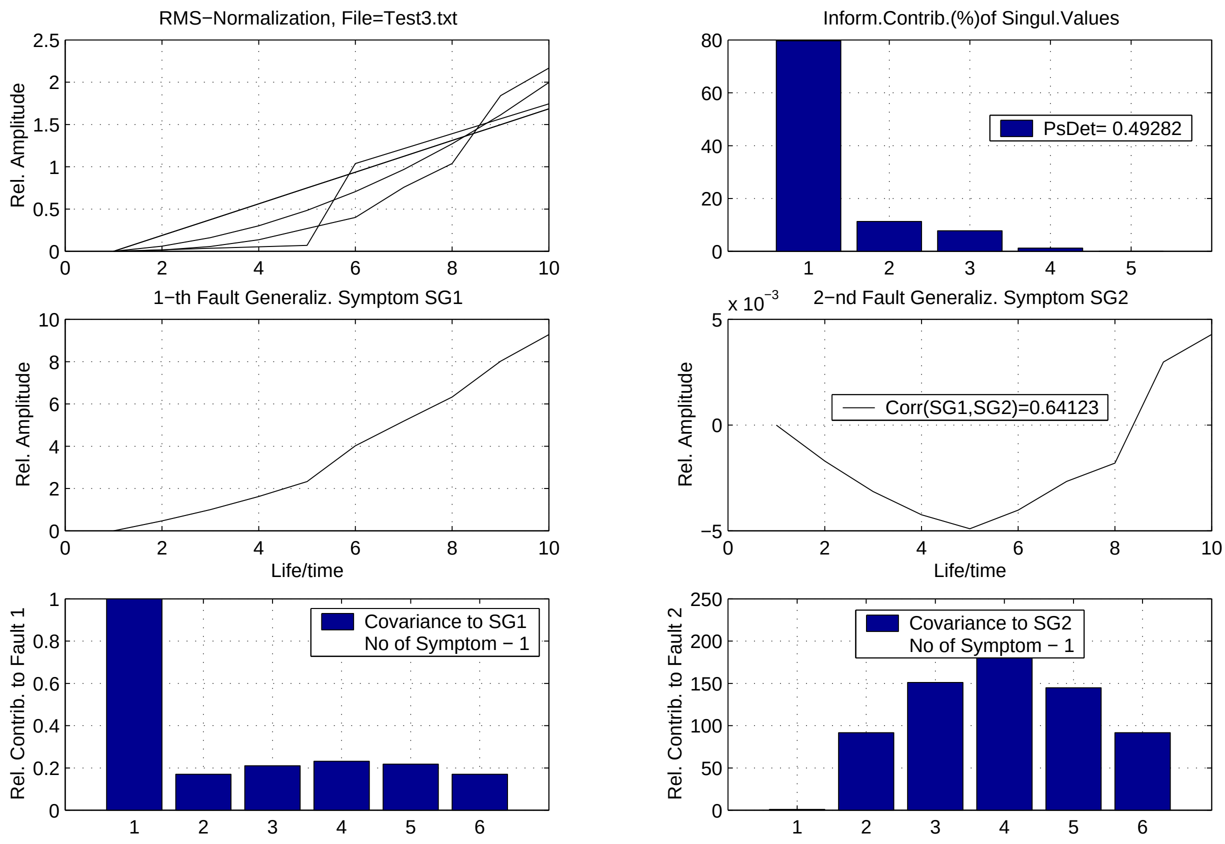

Figure 6: Symptom observation matrix centered and normalized by rms value.

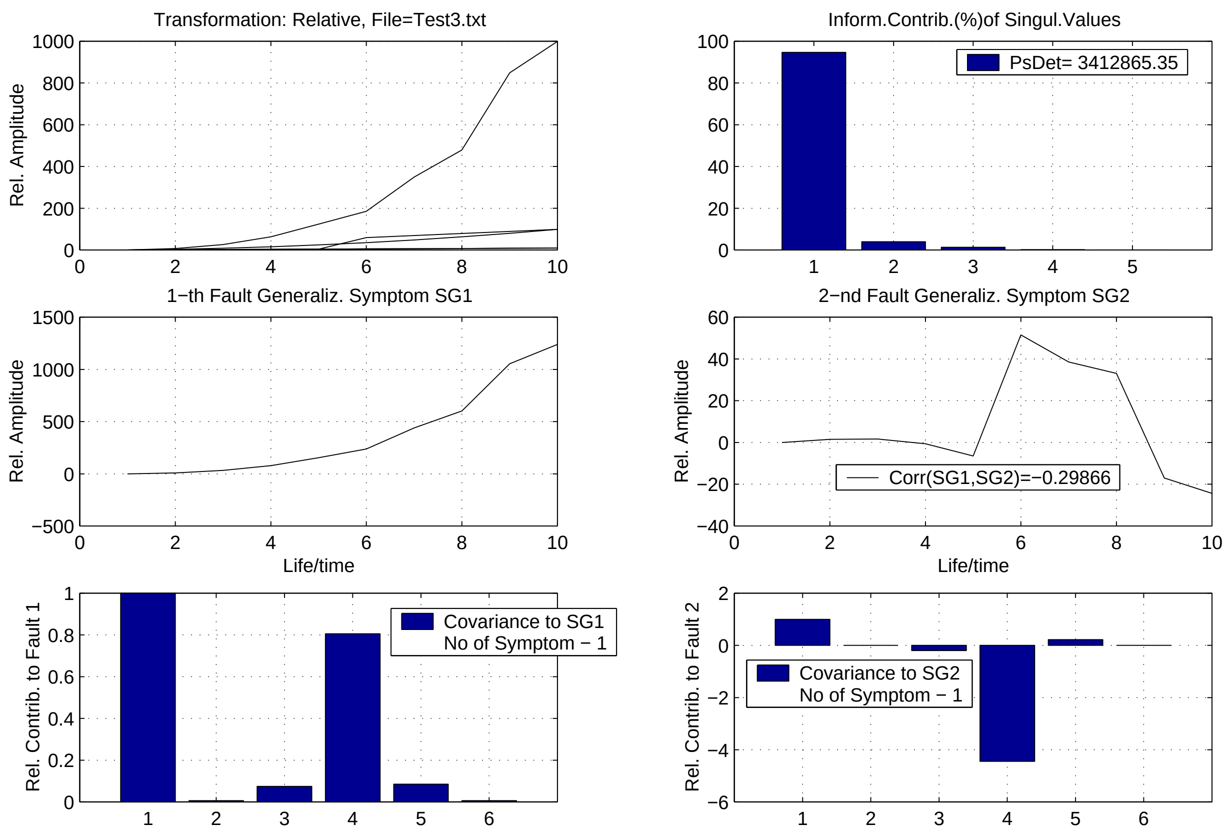

Figure 7: Symptom observation matrix centered and normalized by its initial value.

1. The relative value of symptoms change seems to be the best measure of original symptom characteristics.

2. The value of determinant PsDet must be high, as it reflects the volume of information (fault) space.

3. The range of life evolution of SVD characteristics, $SG1(\theta), SG2(\theta), \sigma_1(\theta), \sigma_2(\theta), D1(\theta), AF(\theta)$, $AS(\theta)$, must be as high as possible after O_{pr} transformation.

The motivation for considering the SVD characteristics in item 1 - 3 is as follows.
Ad1. We must not go to far from the original course of symptoms, as here lies the source of interpreting of the fault and the system damage.
Ad2. The value of determinant of economy SVD, $\mathrm{PsDet}(\theta)$ is nothing more like the volume of the r - dimensional parallepiped of fault space, or the value of information resource, after matrix transformation. From this respect initial value centering and normalization is the best solution.
Ad 3. In order to make inference on system condition with highest confidence, we need the highest range of value evolution of all possible SVD characteristics; of generalized fault: $\mathrm{SG}_t(\theta), \mathrm{SD}_t(\theta), \sigma_t\theta), t = 1, 2..$, as well as combined (or compound) damage characteristics: $\mathrm{PsDet}(\theta), AF(\theta)$, $AS(\theta)$.

As it was shown in [5], in real cases of system symptom condition monitoring we have the dominance of one particular measured symptom, and large σ_1. Hence we will have now the dominance of $\sigma_1(\theta)$, as well as the value of determinant and Frobenius norm of observation matrix will be dominated too, by this large generalized fault. Some better solution may be to use the first order norm of observation matrix.
Using this criteria, and just described way of interpretation of symptom observation matrix, we will exclude matrix centering with column average value, because it is not in the agreement of items 1 - 3 of our criteria. The best transformation keeping well our criteria is centering and normalization with respect of symptom initial value. This is because:

- It takes into account the symptom relative change.
- It preserves the relatively high values of SVD characteristics
- It is the second one transformation extracting the highest number of singular values (after rms normalization).

But it is advised also to use in some cases rms normalization, to look for the highest dimensionality of fault space.

At the end of our general consideration of transformation of symptom observation matrix, let us give the possible meanings to different SVD characteristics. The generalized fault symptoms $SD(\theta)$ and $SG(\theta)$, as the name indicates, they describes the system condition in terms of observed symptoms, as it is indicated by eq 8 and eq 11. Using the singular values $\sigma_t(\theta)$ we can develop another language of system condition description. As one can notice from figure 3, the value of $\sigma_1(\theta)$ for the linear growth of the symptom depends on number of observations. The longer system is in operation, the higher will be the highest reading of $\sigma(\theta)$. This observation is true for every nonzero singular value. What differs the evolution of particular value is its shape, and speed. The most cummulative measures of condition evolution will be the determinant $D1(\theta)$, and Frobenius norm of symptom observation matrix $AF(\theta)$. The first, is a product of singular values, so its speed of growth will escalates rapidly with the life time. The second, Frobenius norm, is the sum of all singular values at given life time, so its value will grow almost linearly.
Summarizing it a little, one can advice to start to look for the general condition of system with cummulative measures of $D1(\theta)$ and $AF(\theta)$. Next when asking 'why such condition has developed'

one can look for the evolution of particular singular values $\sigma_1(\theta)$ or $\sigma_2(\theta)$, and generalized fault symptoms $SG1(\theta)$, $SG2(\theta)$. For some special cases, all this three damage indices maybe similar like in case of one symptom linear growth of figure 2. In this case the originally measured symptom $S_5(\theta)$, and generalized symptom $SG1(\theta)$, first singular value $\sigma_1(\theta)$, $D1(\theta)$ as well as $AF(\theta)$, will behave almost in the same linear way.
It will be of much interest now to see how this measures behave in real situation of systems condition monitoring.

5 SVD-Analysis of some Transformations of the Symptom Observation Matrix

As it was shown in the course of this paper, several computer programs written in MATLAB were developed, named 'svdsymp' with the additional notation of version as the extension of:6, 7, 8, 9, and 0. It was found and described above, that symptom observation matrix centered and normalized initially gives the best results according to our criteria of fault distinguishing with maximum volume of information space. Hence we will use this version of symptom observation matrix transformation, performed by program called 'svdsymp0.m', for further transformation of data taken from vibration condition monitoring of railroad Diesel engines. Here, after each ten thousands kilometers of mileage, vibration process at top of one cylinder were measured and five readings taken. These were vibration acceleration; peak, rms, and average amplitudes, as well as the first two for the vibration velocity. We will show here transformed data of two engines, but many other data sets confirm these findings.
As the first case let us analyze figure 8 showing original not transformed complete history of one running cycle over 230.000 kilometers of mileage, so we have observation matrix with five columns and 23 readings. Vibration symptom course is shown at left top picture with peak vibration acceleration symptom as dominating one. This dominance can be observed also at the top right picture with relative information contribution of singular values, on the shape of 1-th generalized fault symptom, as well as on bottom left picture showing the contribution of originally measured symptoms to SG1. We can see also the life course of 2 -nd generalized fault symptom on the right medium picture, and it does not look as the symptom of some growing fault, but like the oscillating noisy component. It is worth to notice also from the top right picture, that when not transforming the measured data we can only distinguish practically one singular value σ_1, so practically one potential fault $F_1(\theta)$.
Quite another set of fault information can be extracted from the same engine data when we transform them by centering and normalizing to initial values of observed symptom. This situation is shown on figure 9 for the same engine sil24d1, and comparing it to figure 8, one can notice several important differences. At first we can distinguish well at least two singular values σ_1, σ_2, and all five can be noticed from the top right picture. Secondly, centering and normalization to initial symptom values changes completely the course of transformed symptom observation matrix (top left), and all symptoms, even vibration velocity, are giving important contribution to 1-th fault generalized symptom - picture medium left. Also the correlation between SG1 and SG2 is almost zero, and one can say they are really orthogonal. Hence one can say on this example, that idea of transformation of symptom observation matrix is a good one, giving much more fault related data than without transformation of it.
In such a way, every system in operation can be described and analyzed giving us important information concerning the diagnostic value of each symptom, (bottom pictures), and type of damage process developing during system operation, (top right picture). But we can say much more analyzing the evolution of the largest singular values and also composite measures of damage evolution $D1(\theta)$ and $AF(\theta)$. Figure 10 shows the evolution of these fault indices for another Diesel engine sil54d1 of total mileage 250.000km, with the same symptoms being measured. At the top left picture transformed symptom observation matrix is seen, and down on medium left also the first generalized fault symptom $SG1(\theta)$. One can notice that observing $SG1(\theta)$ is really more than observing one of

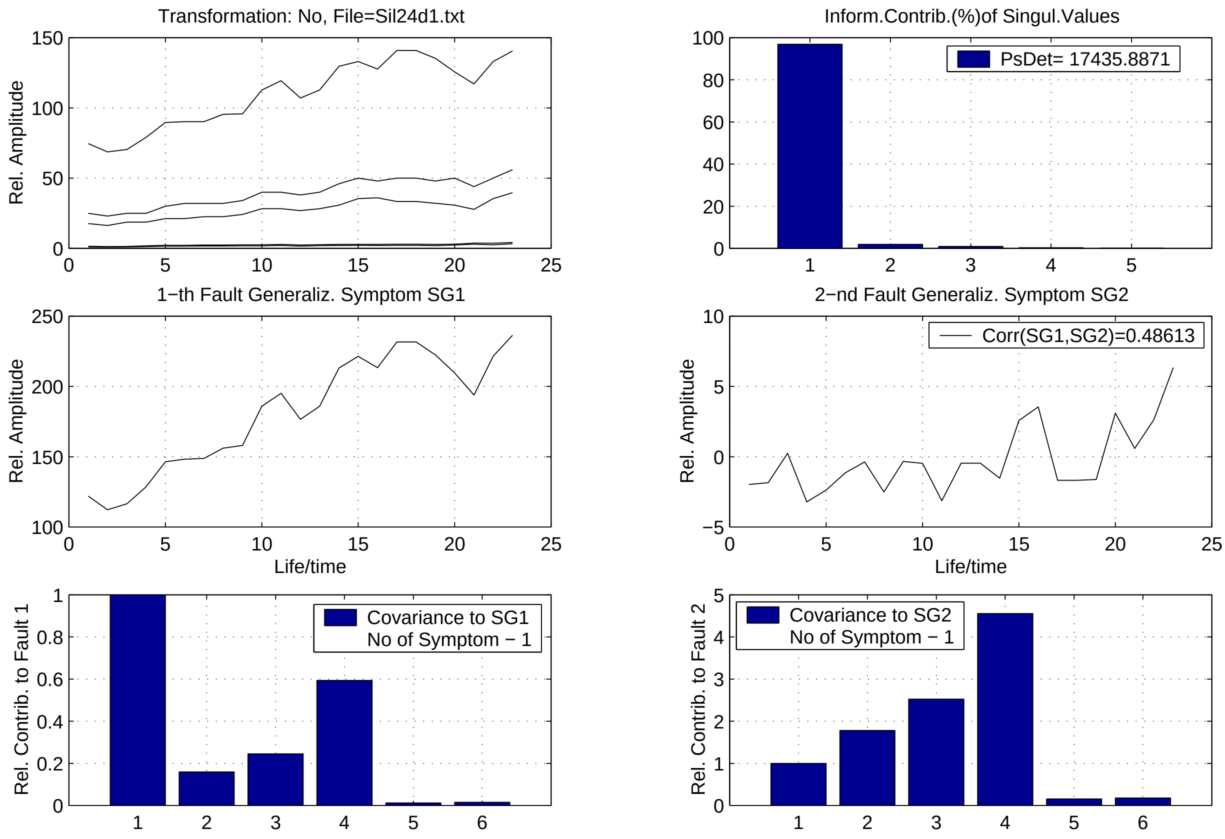

Figure 8: SVD of sil24d1 engine vibration taken as originally received without transformation

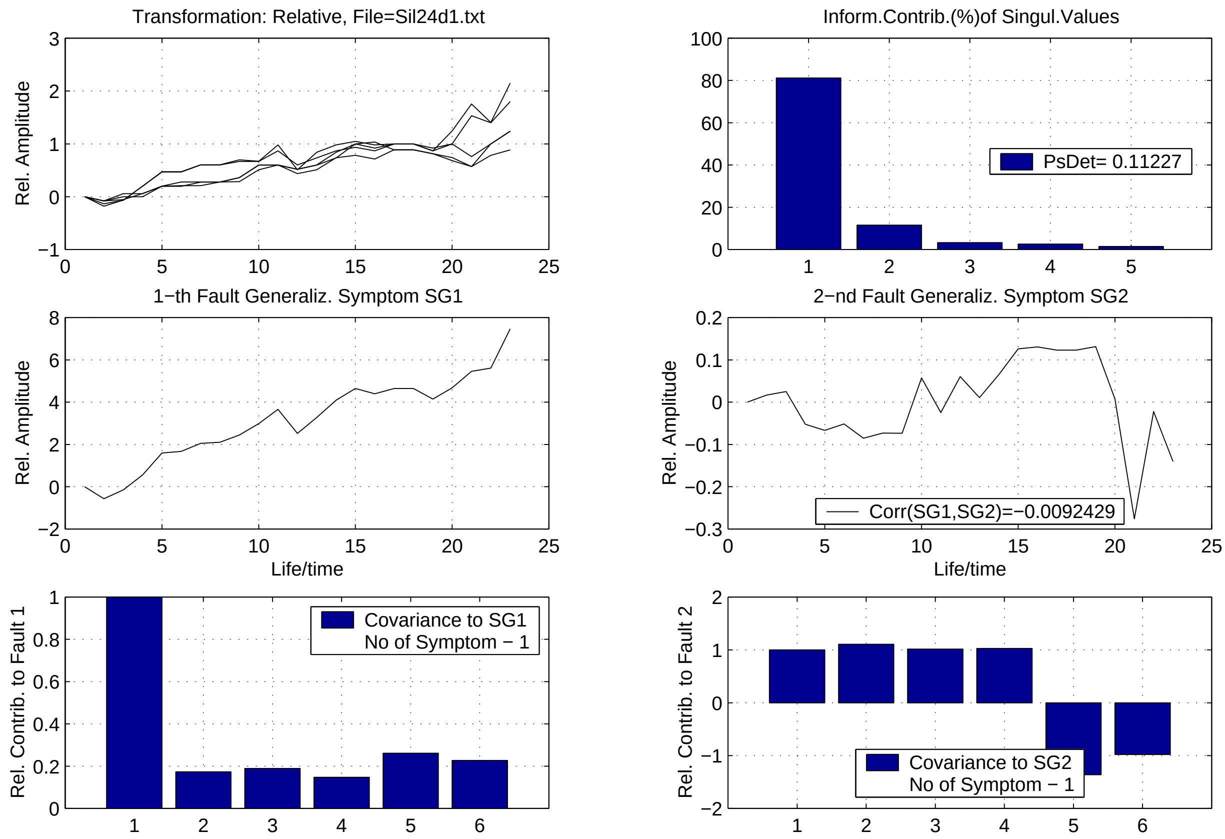

Figure 9: SVD of sil24d1 engine vibration centered and normalized initially

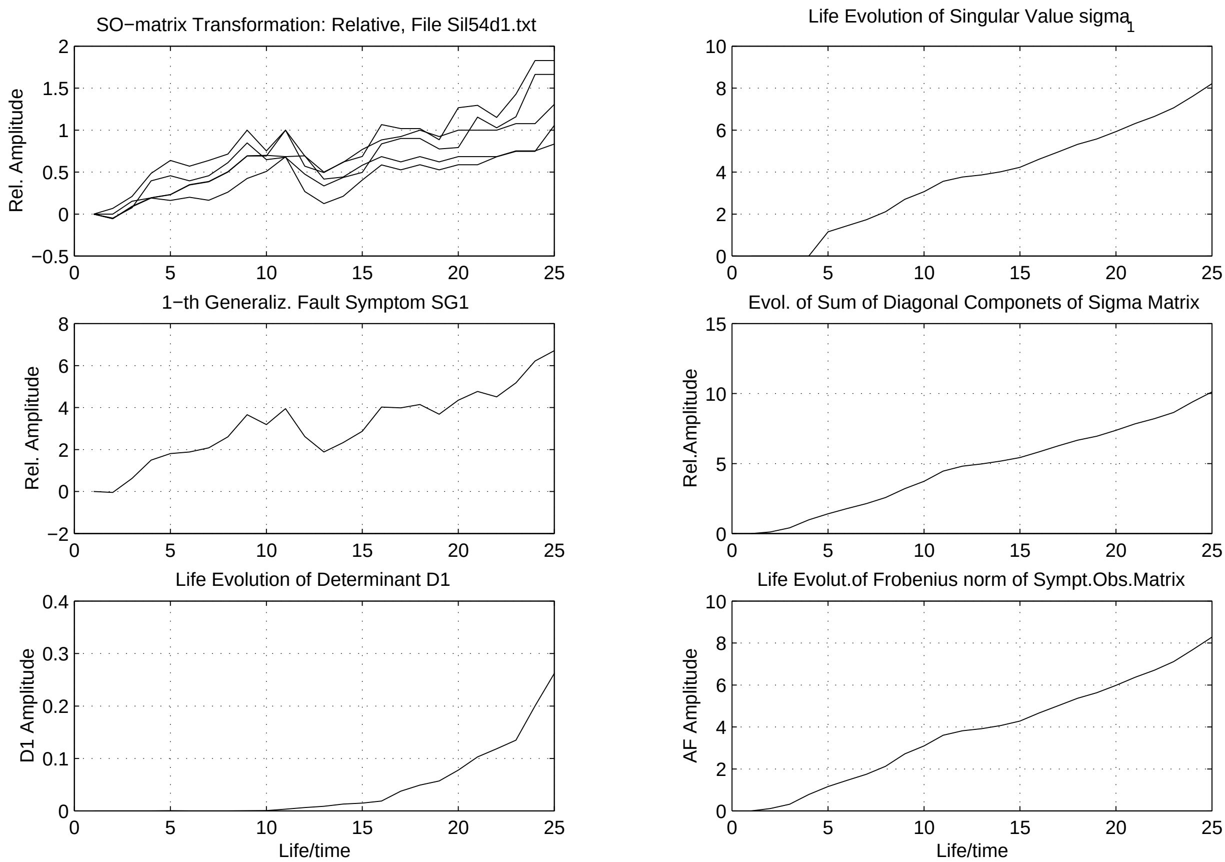

Figure 10: SVD of another engine sil54d1, vibration centered and normalized initially described in terms of singular values and cummulative damage indices

the symptom at the top left, and deserves the name fault generalized symptom. One can notice also that quite another information, smooth and stable, is delivered by the two largest singular values $\sigma_1(\theta)$ and $\sigma_2(\theta)$. Moreover the scale of both singular values differs by one order, having also different course of life evolution. The bottom pictures give us cummulative information about the evolution of all singular values; $\sigma_1, ...\sigma_5$. Here the right picture, the Frobenius norm is more stable, as it is dominated by 1 -th singular value. And left picture of $D1(\theta)$ is sensitive to the evolution of second singular value observing its jump at observation point no 23.

The smoothing property of first singular value, and cummulative damage indices can be used as advantage in cases of nonstable symptom readings. This is the case for example of load sensitive symptoms, as it is shown on figure 11, for vibration condition monitoring of large fan supplying the ear to the deep copper mine. Several vibration symptom were measured here each week and readings are very unstable, - picture top left. More condition related information can be taken from 1-th fault generalized symptom, being also unstable with small oscillation. But the best for condition monitoring seems to be top right picture as the first singular value, and bottom pictures of determinant value D1 (left) and Frobenius norm of observation matrix AF (right). Looking for these composite damage indices it maybe now quite easy to infer on the fan condition, than from the unstable readings of symptoms, or even 1-th fault generalized symptom SG1.

Summarizing the possible application of SVD of symptom observation matrix O_{pr} in reality of condition monitoring, one can see the importance of both ways of SVD description; only by fault generalized symptoms as on Fig 9 , or by singular values $\sigma_1(\theta), \sigma_2(\theta)$ and compound indices like $D1(\theta)$ and $AF(\theta)$. It seems to, that the starting point of each condition monitoring application should be left part of figure 10, with the graphic outline of symptom observation matrix, first fault generalized

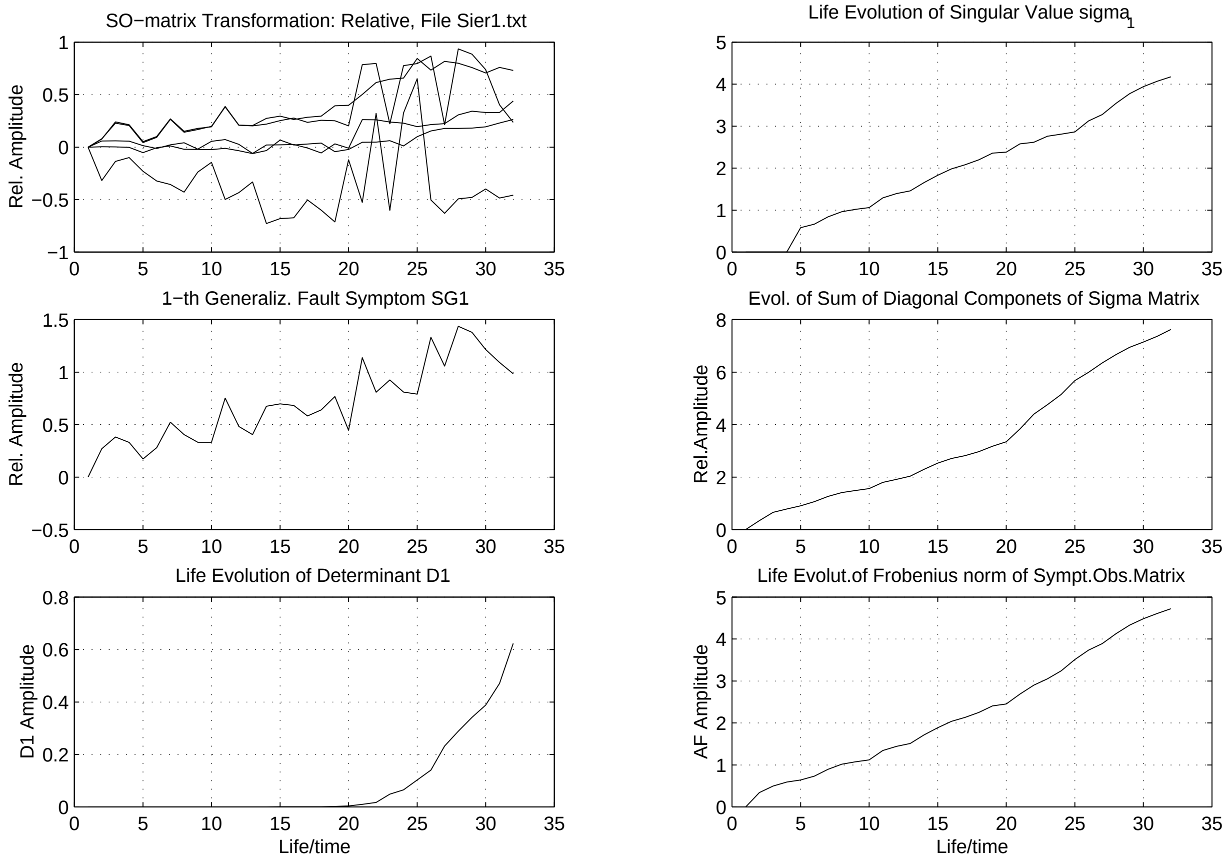

Figure 11: SVD description of huge deep mining fan with vibration symptoms sensitive to the load

symptom SG1, current value of determinant $D1(\theta)$, and the Frobenius norm $AF(\theta)$. The last two damage indices can be presented with its limit values, calculated for example by Neuman - Pearson rule [1], [2]. The detailed investigation of fault evolution and relative importance of measured symptom can be more clear when taken into account bottom pictures of figure 9.
In the same way of reasoning one can say, that application of SVD to systems condition monitoring can accomplish much more, instead looking for one symptom evolution as it was before. Now we can calculate and monitor several damage indices concerning with one generalized fault; like generalized fault symptoms SG_t or singular values σ_t, and the the value of determinant D1 or Frobenius norm AF of symptom observation matrix. The particular aptitude to particular damage measure can be system dependent, and sometimes even mission dependent. So it must be studied separately in each particular case of given application.

6 Summary

Singular value decomposition applied to symptom observation matrix used in systems condition monitoring were studied in this paper. Basing on previous authors developments of this method, several computer programs were elaborated and applied for the simulated data as well as for real vibration condition monitoring of Diesel engines and industrial fans. It was found that the symptom observation matrix is valuable resource of information concerning systems damage evolution and / or modification. It was found too, that among many possible methods of symptom observation matrix transformation, the centering and normalization to initial symptom value gives the greatest advantages distinguishing much more types of system wear / modification than without transformation. It allows to study the creation of newly found generalized fault symptoms, measures, and indices, and maximize the amount of condition related information. It will allow also to find some new meanings and application to singular values, as well to compound damage measure $D1(\theta)$ and $AF(\theta)$.

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